degradation in rendezvous position accuracy and a modest degradation in rendezvous velocity accuracy.

Acknowledgments

This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract NAS 7-100, sponsored by the National Aeronautics and Space Administration.

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Technical Comments

AIAA 81-4249

Comment on "Stability of a Precision Attitude Determination Scheme"

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IN Ref. 1, the stability of an attitude determination scheme discussed in Ref. 2 is investigated. The purposes of this Comment are 1) to show that certain conclusions reached in Ref. 1 are incorrect, and 2) to show that other conclusions, while correct, can be deduced more simply.

By way of introduction, consider a spacecraft which is nominally fixed in orientation relative to inertial space. Let ψ denote the rotation angle of the spacecraft about an inertially fixed axis and let ω denote the angular rate of the spacecraft about a nominally coincident body-fixed axis. Then

$$\dot{\psi} \approx \omega \tag{1}$$

Assume that the gyroscopically measured angular rate about the body-fixed axis can be modeled as

$$\omega_g = \omega - d - \eta_v \tag{2}$$

where

$$\dot{d} = \eta_u \tag{3}$$

and η_v and η_u are statistically stationary zero-mean white noise processes. The spacecraft dynamics about this body-fixed axis may be modeled as

$$I\dot{\omega} \approx T_c + T_d \tag{4}$$

where I, T_c, and T_d denote the spacecraft moment of inertia, the control torque, and the disturbance torque about this axis, respectively. If the control torque is chosen to be

$$T_c = -k(\omega_g - u) \tag{5}$$

where u is the commanded angular rate, then by choosing k sufficiently large, the difference $\omega_g - u$ could be made arbitrarily small, were it not for the white noise term in Eq. (2). This can be seen by substituting Eqs. (2) and (5) into Eq. (4). The approximation

$$\omega_g \approx u \tag{6}$$

is often made despite the white noise term in Eq. (2), yielding

$$\dot{\psi} \approx d + u + \eta_v \tag{7}$$

Equations (3) and (7) are equivalent to Eq. (1) in Ref. 1 and Eqs. (1) and (3) in Ref. 2. The purpose of this derivation has been to establish the assumptions and approximations typically associated with these equations.

It is argued in Ref. 1 that the fourth-order system consisting of Eqs. (3) and (7) and analogous equations for propagating optimal estimates of ψ and d (based upon discrete noisy measurements of ψ) is not stable. This conclusion is correct, but the sequence of eighteen equations used to reach this conclusion is not necessary. It follows from Eq. (3) that the variance of d increases linearly with time. Thus, if system instability in a stochastic problem is interpreted to mean that certain covariance matrix elements grow without bound as time becomes large, any system of equations including Eq. (3) is unstable.

In addition, certain arguments used in the derivation of this result in Ref. 1 are not correct. In particular, if A, B, and H denote system dynamics, control distribution, and measurement distribution matrices for a time-invariant linear system, controllability of the pair (A,B) is not a necessary condition for existence of a stable solution to the infinite time regulator problem. Nor is observability of the pair (A,H) necessary for existence of a stable solution to the infinite time estimator problem. Both these conditions are elements of sets of sufficient conditions for stability,[3] but are not necessary. If, for example, the term $-d/\tau$ $(\tau>0)$ were added to the right-hand side of Eq. (3), the system of Eqs. (3) and (7) (ignoring the white noise terms) would remain uncontrollable, but could be stabilized by a suitable choice of u.

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It is also concluded in Ref. 1 that the attitude determination accuracy will degrade with time, due to the instability of the combined estimator-regulator system. This conclusion is not correct, and is contradicted by comments following Eq. (9). Though elements of the state covariance matrix (and the state estimate covariance matrix) will grow without bound as time becomes large, the elements of the state estimate error covariance matrix will remain finite. This is guaranteed by the fact that the two-state-variable system is observable with measurements of ψ and the state is controllable by the process noise.[3]

It is finally concluded in Ref. 1 that this instability of the combined estimator-regulator problem imposes design constraints on the attitude control system, in the sense that the mission will end prematurely if the attitude and gyro drift rate diverge too rapidly. This is not really the case, however. The apparent instability arises only because Eqs. (3) and (7) are extremely simple representations of spacecraft and gyro dynamics. While perhaps not unreasonable over a short time interval, this model is unrealistic over long periods of time. A simple modification of the model involves addition of the term $-d/\tau$ to the right-hand side of Eq. (3). The consequent modeling of gyro drift as a first-order Gauss-Markov process with correlation time τ, rather than as a random walk, eliminates the instability problem and typically has a negligible effect on the optimum estimator gains, for large τ. The key point is that a satisfactory steady-state estimator can be derived from a model which is simple and is reasonably accurate only over short periods of time.

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Reply by Author to L.J. Wood

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THE author is thankful to Dr. Wood for his valued comments. It is rightly pointed out that the instability of the state covariance is due to the particular choice of the system model. The model is approximate but has the advantage of easy on-board implementation. Since attitude covariance convergence is not assured directly, this system model fails to bring out the long-term performance of the attitude determination scheme. The use of so many equations to show the state covariance divergence is to find quantitatively the amount of uncertainty expected of such a system model at any given instant of time.

Even if a first-order Gauss-Markov process is used instead of a white noise to represent the random change in the gyro bias drift rate, the divergence of the attitude covariance cannot be got rid of, since the system matrix pair (A,B) is still neither controllable nor stabilizable with drift rate feedback control alone.

In order to ensure covergence of the attitude covariance, the response of the attitude control system may be modelled with a first-order lag, besides the gyro bias drift rate as a first-order Gauss-Markov process.

$$\dot{\psi} = -\psi/\tau_1 + d + u + \eta_v \qquad d = -d/\tau_2 + \eta_u$$

where both τ_1 and τ_2 are large compared to the filter update interval T.

The system is still not controllable, but it is now stabilizable with drift rate feedback alone. The attitude covariance of the combined regulator-estimator now remains bounded as $t \to \infty$. Analytical results on the long-term performance of the algorithm with the modified model can then be derived.

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Comment on "Orbital Decay Due to Drag in an Exponentially Varying Atmosphere"

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THE paper by A.J.M. Chakravarty[1] describes the formal application of the two-variable Asymptotic Expansion Procedure (AEP) to the problem of two-dimensional orbital motion of a ballistic vehicle as perturbed by aerodynamic interaction with a variable-density atmosphere. The purpose of this Technical Comment is to help place the reported study in a somewhat wider context, and thereby draw attention to some interesting and hopefully useful results obtained in earlier studies on the same subject. Specifically, the wider context sought is that of other, earlier, and formal applications of the multivariable AEP to the problem of aerodynamically perturbed satellite motion.

A special case of the multivariable AEP is the two-variable AEP (used in Ref. 1), developed by Kevorkian[2] (see also Refs. 4 and 6). Here, two linear time-like "clocks" are used: a "fast clock" $\tau_1 \triangleq \tau$, and a "slow clock" $\tau_2 \triangleq \epsilon\tau$ $(0 \leq \epsilon \ll 1)$, where τ represents the independent variable. In studies of two-dimensional, aerodynamically perturbed satellite motion the independent variable typically represents the central angle (between a suitable in-plane inertial reference vector and the radius vector), whereas the small parameter ϵ is usually defined as the ratio of drag to weight at initial time.

Kevorkian applied the two-variable AEP to the problem of two-dimensional, aerodynamically perturbed motion of a ballistic satellite in a constant-density atmosphere[2] (see also Ref. 3, p. 3). Kevorkian's problem formulation was then generalized by Simmons,[3] who included the effects of aerodynamic lift on the satellite orbit (see also Ref. 4, pp. 264-

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†Where "C" indicates Chakravarty's equations in Ref. 1.